We use substitution

$$u = Rkt + 1 \tag{4.12}$$

$$\frac{du}{dt} = Rk \tag{4.13}$$

$$dt = \frac{1}{Rk} du \tag{4.14}$$

To get

$$P_H(t) = \frac{1}{k} \int \frac{1}{u} du \tag{4.15}$$

$$P_H(t) = \frac{1}{k} \ln(Rkt + 1) + c \Big|_0^t \tag{4.16}$$

Using initial conditions

$$P_H(0) = 0 \tag{4.17}$$

We find

$$c = 0 \tag{4.18}$$

Therefore the skydivers horizontal position, P_H, at time t is given by

$$P_H(t) = \frac{1}{k} \ln(Rkt + 1) \tag{4.19}$$

Vertical component

We perform the same calculations as above
Total acceleration is the acceleration due to drag (in the positive direction) minus acceleration due to gravity (in the negative direction):

$$\frac{dv_V(t)}{dt} = -g + kv_V(t)^2 \tag{4.20}$$

We perform a separation of variables and rearrange for velocity

$$-\frac{\tanh^{-1}\left(\sqrt{\frac{k}{g}} v_V(t)\right)}{\sqrt{gk}} = t + c \tag{4.21}$$

$$v_V(t) = -\sqrt{\frac{g}{k}} \tanh\left(\sqrt{gk}(t + c)\right) \tag{4.22}$$

We note that $\sqrt{\frac{g}{k}} = v_T$ and see the velocity slowly approaches terminal velocity as $t \to \infty$ as expected

We use initial conditions to find c=0 and integrate to find position as a function of time

$$P_V(t) = -\frac{1}{k}\ln\left(\cosh\left(\sqrt{gk}\cdot t\right)\right) \tag{4.23}$$

We therefore have:

Horizontal:

$$v_H(t) = \frac{R}{Rkt+1} \tag{4.24}$$

$$P_H(t) = \frac{1}{k}\ln(Rkt+1) \tag{4.25}$$

$$t = \frac{e^{kP_H}-1}{Rk} \tag{4.26}$$

Vertical:

$$v_V(t) = -v_T\tanh\left(\sqrt{gk}\cdot t\right) \tag{4.27}$$

$$P_V(t) = -\frac{1}{k}\ln\left(\cosh\left(\sqrt{gk}\cdot t\right)\right) \tag{4.28}$$

$$t = \frac{\cosh^{-1}\left(e^{-kP_V}\right)}{\sqrt{gk}} \tag{4.29}$$